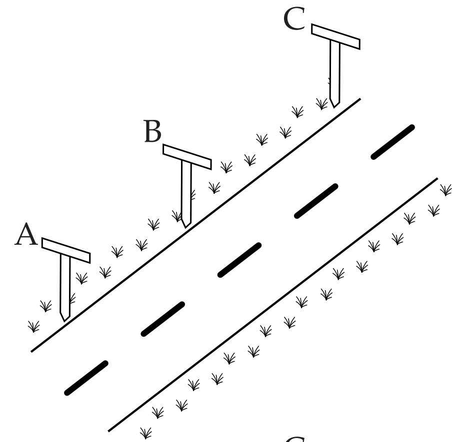
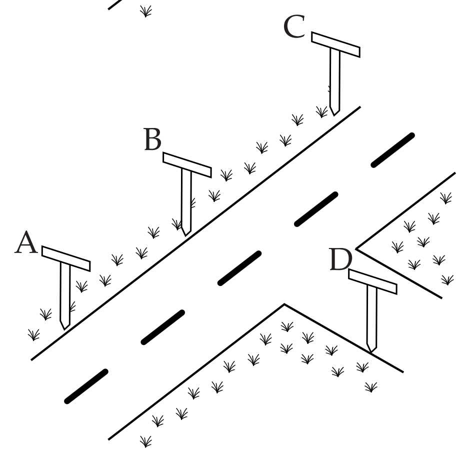
[SQA] 1. (*a*) Roadmakers look along the tops of a set of T-rods to ensure that straight sections of road are being created. Relative to suitable axes the top left corners of the T-rods are the points $A(-8,-10,-2)$, $B(-2,-1,1)$ and $C(6,11,5)$.

Determine whether or not the section of road ABC has been built in a straight line. **3**



(*b*) A further T-rod is placed such that D has coordinates $(1,-4,4)$.

Show that DB is perpendicular to AB. **3**



Part	Marks	Level	Calc.	Content	Answer	U3 OC1
(*a*)	3	C	CN	G23	the road ABC is straight	2001 P1 Q3
(*b*)	3	C	CN	G27, G17	proof	

•[1] ic: interpret vector (e.g. $\overrightarrow{AB}$)
•[2] ic: interpret multiple of vector
•[3] ic: complete proof

•[4] ic: interpret vector (i.e. $\overrightarrow{BD}$)
•[5] ss: state requirement for perpend.
•[6] ic: complete proof

•[1] e.g. $\overrightarrow{AB} = \begin{pmatrix} 6 \\ 9 \\ 3 \end{pmatrix}$

•[2] e.g. $\overrightarrow{BC} = \begin{pmatrix} 8 \\ 12 \\ 4 \end{pmatrix} = \frac{4}{3}\overrightarrow{AB}$ **or**

$\overrightarrow{AB} = 3\begin{pmatrix} 2 \\ 3 \\ 1 \end{pmatrix}$ and $\overrightarrow{BC} = 4\begin{pmatrix} 2 \\ 3 \\ 1 \end{pmatrix}$

•[3] a common direction exists **and** a common point exists, so A, B, C collinear

•[4] $\overrightarrow{BD} = \begin{pmatrix} 3 \\ -3 \\ 3 \end{pmatrix}$

•[5] $\overrightarrow{AB}.\overrightarrow{BD} = 0$
•[6] $\overrightarrow{AB}.\overrightarrow{BD} = 18 - 27 + 9 = 0$

or

•[5] $\overrightarrow{AB}.\overrightarrow{BD} = 18 - 27 + 9$
•[6] $\overrightarrow{AB}.\overrightarrow{BD} = 0$ so AB is at right angles to

[SQA] 2. (a) Show that the points L(–5, 6, –5), M(7, –2, –1) and N(10, –4, 0) are collinear. 4

(b) Find the ration in which M divides LN. 1

Part	Marks	Level	Calc.	Content	Answer	U3 OC1
(*a*)	4	C	CN	G23		1991 P1 Q7
(*b*)	1	C	CN	G25		

•[1] $\overrightarrow{LM} = \begin{pmatrix}12\\-8\\4\end{pmatrix}$ or equivalent combinations for (a)

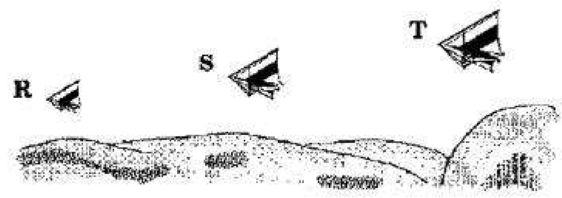
•[2] $\overrightarrow{MN} = \begin{pmatrix}3\\-2\\1\end{pmatrix}$

•[3] $\overrightarrow{LM} = 4\overrightarrow{MN}$

•[4] vectors are parallel and have common point so L, M, N are collinear

•[5] 4:1

[SQA] 3. Relative to the top of a hill, three gliders have positions given by R(–1, –8, –2), S(2, –5, 4) and T(3, –4, 6).
Prove that R, S and T are collinear. 3



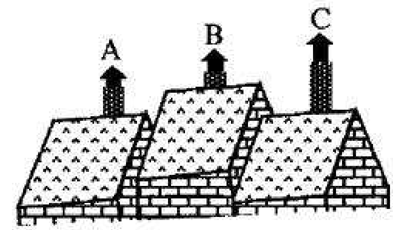
Part	Marks	Level	Calc.	Content	Answer	U3 OC1
	3	C	CN	G23		1994 P1 Q4

•[1] $\overrightarrow{ST} = \begin{pmatrix}1\\1\\2\end{pmatrix}$ or equivalent **and** $\overrightarrow{RS} = \begin{pmatrix}3\\3\\6\end{pmatrix}$ or equivalent

•[2] $\overrightarrow{RS} = 3\overrightarrow{ST}$ or equiv.

•[3] RS // ST **and** S is common.

[SQA] 4. Relative to a suitable set of axes, the tops of three chimneys have coordinates given by A(1, 3, 2), B(2, −1, 4) and C(4, −9, 8).
Show that A, B and C are collinear. 3



Part	Marks	Level	Calc.	Content	Answer	U3 OC1
	3	C	CN	G23		1997 P1 Q2

•[1] $\overrightarrow{AB} = \begin{pmatrix} 1 \\ -4 \\ 2 \end{pmatrix}$

•[2] $\overrightarrow{BC} = \begin{pmatrix} 2 \\ -8 \\ 4 \end{pmatrix}$ **AND** $\overrightarrow{BC} = 2 \times \overrightarrow{AB}$

•[3] $\overrightarrow{AB} \,||\, \overrightarrow{BC}$ & B is common hence A, B, C collinear

[SQA] 5. Show that $P(2, 2, 3)$, $Q(4, 4, 1)$ and $R(5, 5, 0)$ are collinear and find the ratio in which Q divides PR. **4**

Part	Marks	Level	Calc.	Content	Answer	U3 OC1
	4	C	CN	G23, G25	$\overrightarrow{QR} = \frac{1}{2}\overrightarrow{PQ}$, $PQ : QR = 2 : 1$	1990 P1 Q4

•[1] $\overrightarrow{PQ} = \begin{pmatrix} 2 \\ 2 \\ -2 \end{pmatrix}$

•[2] $\overrightarrow{QR} = \begin{pmatrix} 1 \\ 1 \\ -1 \end{pmatrix} = \frac{1}{2}\overrightarrow{PQ}$

or equivalent

•[3] vectors parallel and have pt in common so pts collinear

•[4] $PQ{:}QR = 2{:}1$

[SQA] 6. A is the point $(2, -5, 6)$, B is $(6, -3, 4)$ and C is $(12, 0, 1)$. Show that A, B and C are collinear and determine the ratio in which B divides AC. **4**

Part	Marks	Level	Calc.	Content	Answer	U3 OC1
	4	C	CN	G23, G25	2 : 3 or equivalent	1996 P1 Q6

•[1] $\overrightarrow{AB} = \begin{pmatrix} 4 \\ 2 \\ -2 \end{pmatrix}$ *or* $\overrightarrow{AC} = \begin{pmatrix} 10 \\ 5 \\ -5 \end{pmatrix}$ *or* $\overrightarrow{BC} = \begin{pmatrix} 6 \\ 3 \\ -3 \end{pmatrix}$

•[2] $\overrightarrow{AB} = 2\begin{pmatrix} 2 \\ 1 \\ -1 \end{pmatrix}$ *and* $\overrightarrow{BC} = 3\begin{pmatrix} 2 \\ 1 \\ -1 \end{pmatrix}$ or equivalent

•[3] $AB \,||\, BC$ **and B is point in common**

•[4] 2:3 (*or* equivalent e.g. $1{:}1\frac{1}{2}$)

[SQA] 7. D, E and F have coordinates $(10, -8, -15)$, $(1, -2, -3)$ and $(-2, 0, 1)$ respectively.

(*a*) (i) Show that D, E and F are collinear.

(ii) Find the ratio in which E divides DF. **4**

(*b*) G has coordinates $(k, 1, 0)$.

Given that DE is perpendicular to GE, find the value of k. **4**

Part	Marks	Level	Calc.	Content	Answer	U3 OC1
(*a*)	4	C	CN	G23, G24	3 : 1	2009 P1 Q22
(*b*)	4	C	CN	G27	$k = 7$	

•[1] ss: use vector approach	•[1] $\overrightarrow{\mathrm{DE}} = \begin{pmatrix} -9 \\ 6 \\ 12 \end{pmatrix}$ or $\overrightarrow{\mathrm{EF}} = \begin{pmatrix} -3 \\ 2 \\ 4 \end{pmatrix}$
•[2] ic: compare two vectors	•[2] 2nd column vector and $(\overrightarrow{DE}) = 3\overrightarrow{\mathrm{EF}}$
•[3] ic: complete proof	•[3] $\overrightarrow{\mathrm{DE}}$ and $\overrightarrow{\mathrm{EF}}$ have common point and common direction; hence D, E and F are collinear
•[4] ic: state ratio	•[4] 3 : 1
•[5] ss: use vector approach	•[5] $\overrightarrow{\mathrm{GE}} = \begin{pmatrix} 1-k \\ -3 \\ -3 \end{pmatrix}$
•[6] ss: know scalar product $=0$ for $\perp$ vectors	•[6] $\overrightarrow{\mathrm{DE}}.\overrightarrow{\mathrm{GE}} = 0$
•[7] pd: start to solve	•[7] $-9(1-k) + 6 \times (-3) + 12 \times (-3)$
•[8] pd: complete	•[8] $k = 7$

[SQA] 8. ABCD is a quadrilateral with vertices $A(4,-1,3)$, $B(8,3,-1)$, $C(0,4,4)$ and $D(-4,0,8)$.

(*a*) Find the coordinates of M, the midpoint of AB. **1**

(*b*) Find the coordinates of the point T, which divides CM in the ratio 2 : 1. **3**

(*c*) Show that B, T and D are collinear and find the ratio in which T divides BD. **4**

Part	Marks	Level	Calc.	Content	Answer	U3 OC1
(*a*)	1	C	CN	G6, G25		1989 P2 Q2
(*b*)	3	C	CN	G25		
(*c*)	4	C	CN	G23, G25		

(*a*) $\bullet^1$ $(6,1,1)$

(*b*) $\bullet^2$ *e.g.* $\overrightarrow{CM}=\begin{pmatrix}6\\-3\\-3\end{pmatrix}$

$\bullet^3$ $\overrightarrow{CT}=\begin{pmatrix}4\\-2\\-2\end{pmatrix}$

$\bullet^4$ $T=(4,2,2)$

(*c*) $\bullet^5$ *e.g.* $\overrightarrow{BT}=\begin{pmatrix}-4\\-1\\3\end{pmatrix}$

$\bullet^6$ $\overrightarrow{TD}=\begin{pmatrix}-8\\-2\\6\end{pmatrix}=2\times\overrightarrow{BT}$

$\bullet^7$ TD is parallel to BT, T is common point so B, T, D collinear

$\bullet^8$ BT : TD = 1 : 2

[END OF QUESTIONS]